Schwinger Mechanism for Gluon Pair Production in the Presence of Arbitrary Time Dependent Chromo-Electric Field

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(Dated: January 8, 2009)

Abstract

We study Schwinger mechanism for gluon pair production in the presence of arbitrary time-dependent chromo-electric background field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3) by directly evaluating the path integral. We obtain an exact expression for the probability of non-perturbative gluon pair production per unit time per unit volume and per unit transverse momentum $\frac{dW}{d^4x d^2 p_T}$ from arbitrary $E^a(t)$. We show that the tadpole (or single gluon) effective action does not contribute to the non-perturbative gluon pair production rate $\frac{dW}{d^4x d^2 p_T}$. We find that the exact result for non-perturbative gluon pair production is independent of all the time derivatives $\frac{d^n E^a(t)}{dt^n}$ where $n = 1, 2, ...\infty$ and has the same functional dependence on two casimir invariants $[E^a(t)E^a(t)]$ and $[d_{abc}E^a(t)E^b(t)E^c(t)]^2$ as the constant chromo-electric field E^a result with the replacement: $E^a \to E^a(t)$. This result may be relevant to study the production of a non-perturbative quark-gluon plasma at RHIC and LHC.

PACS numbers: PACS: 11.15.-q, 11.15.Me, 12.38.Cy, 11.15.Tk

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Typeset by REVTEX   1

I. INTRODUCTION

An exact non-perturbative result for electron-positron pair production from a constant electric field was obtained by Schwinger in 1951 by using proper time method [1]. In QCD this result depends on two independent casimir/gauge invariants $C_1 = [E^a E^a]$ and $C_2 = [d_{abc} E^a E^b E^c]^2$ with color indices a, b, c=1,2,...8 in SU(3) [2]. Recently we have extended this calculation to arbitrary time dependent electric field $E(t)$ in QED [3] and to arbitrary time dependent chromo-electric field $E^a(t)$ in QCD for quark-antiquark case [4]. This result relies crucially on the validity of the shift conjecture, which has not yet been established. In this paper we will extend this calculation to study non-perturbative gluon pair production from arbitrary $E^a(t)$. Unlike constant field E^a case [2], we encounter a non-vanishing single gluon (or tadpole) term in the presence of time dependent chromo-electric field $E^a(t)$. However, we show that the non-perturbative tadpole effective action $\frac{dS_{\text{tad}}}{d^4 x d^2 p_T}$ contains $\delta^{(2)}(\vec{p}_T)$ distribution and hence, for any non-vanishing transverse momentum, it does not contribute to non-perturbative gluon pair production rate $\frac{dW}{d^4 x d^2 p_T}$ via Schwinger mechanism. This result may be relevant to study the production of a non-perturbative quark-gluon plasma at RHIC and LHC [2, 5, 6, 7].

We obtain the following exact non-perturbative result for the probability of gluon (pair) production per unit time, per unit volume and per unit transverse momentum from an arbitrary time dependent chromo-electric field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3):

$$\frac{dW_{g(\bar{g})}}{dt d^3 x d^2 p_T} = \frac{1}{4\pi^3} \sum_{j=1}^{3} |g\Lambda_j(t)| \ln[1 + e^{-\frac{\pi p_T^2}{|g\Lambda_j(t)|}}]. \tag{1}$$

In the above equation

$$\Lambda_1^2(t) = \frac{C_1(t)}{2}[1 - \cos\theta(t)]; \quad \Lambda_{2,3}^2(t) = \frac{C_1(t)}{2}[1 + \cos(\frac{\pi}{3} \pm \theta(t))]; \quad \cos^3\theta(t) = -1 + 6C_2(t)/C_1^3(t) \tag{2}$$

where

$$C_1(t) = [E^a(t) E^a(t)]; \qquad \text{and} \qquad C_2(t) = [d_{abc} E^a(t) E^b(t) E^c(t)]^2 \tag{3}$$

are two independent time-dependent casimir/gauge invariants in SU(3).

This result has the remarkable feature that it is independent of all the time derivatives $\frac{d^n E^a(t)}{dt^n}$ and has the same functional form as the constant chromo-electric field E^a result [3] with: $E^a \rightarrow E^a(t)$.

We will present a derivation of eq. (1) in this paper.

II.   SCHWINGER MECHANISM IN QCD IN THE PRESENCE OF ARBITRARY TIME-DEPENDENT CHROMO-ELECTRIC FIELD

In the background field method of QCD [8, 9] the gauge field is the sum of classical chromo-field A_μ^a and the quantum gluon field Q_μ^a. The non-abelian field tensor becomes

$$F_{\mu\nu}^a[A+Q] = \partial_\mu(A_\nu^a+Q_\nu^a) - \partial_\nu(A_\mu^a+Q_\mu^a) + gf^{abc}(A_\mu^b+Q_\mu^b)(A_\nu^c+Q_\nu^c). \tag{4}$$

The gauge field Lagrangian density is

$$\mathcal{L}_{gl} = -\frac{1}{4}F_{\mu\nu}^a[A+Q]F^{\mu\nu a}[A+Q] - \frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 \tag{5}$$

where the second term in the right hand side is the gauge fixing term [8, 9]. The covariant derivative is given by

$$D_\mu^{ab}[A] = \delta^{ab}\partial_\mu + gf^{abc}A_\mu^c. \tag{6}$$

Keeping terms up to quadratic in Q field (for gluon pair production) and using Feynman-t' Hooft gauge (α=1) we find from eq. (5)

$$\int d^4x \mathcal{L} = \frac{1}{2}\int d^4x \; [-(D_\mu[A]Q_\nu^a)F^{\mu\nu a}[A] + Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b}]$$
$$= \frac{1}{2}\int d^4x \; [(D_\mu[A]F^{\mu\nu a}[A])Q_\nu^a + Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b}] \tag{7}$$

where

$$M_{\mu\nu}^{ab}[A] = g_{\mu\nu}[D^2(A)]^{ab} - 2gf^{abc}F_{\mu\nu}^c[A] \tag{8}$$

with $g_{\mu\nu} = (1, -1, -1, -1)$.

The vacuum-to-vacuum transition amplitude for gluon (we will consider the ghost later in the derivation) in the presence of classical chromo-field A_μ^a is given by

$$< 0|0 >^A = \frac{Z[A]}{Z[0]} = \frac{\int [dQ]e^{i\int d^4x[Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b}+(D_\mu[A]F^{\mu\nu a}[A])Q_\nu^a]}}{\int [dQ]e^{i\int d^4x Q^{\mu a}M_{\mu\nu}^{ab}[0]Q^{\nu b}}}. \tag{9}$$

We choose the arbitrary time-dependent chromo-electric field $E^a(t)$ to be along the $z-$axis (the beam direction) and work in the choice $A_3^a = 0$ [4] so that

$$A_\mu^a(x) = -\delta_{\mu 0} E^a(t) z. \tag{10}$$

The color indices $a=1,2,...8$ are arbitrary. From eq. (10) we find

$$D_\mu[A] F^{\mu\nu a}[A] = \frac{dE^a(t)}{dt} \, \delta_3^\nu. \tag{11}$$

Hence the single gluon term $(D_\mu[A] F^{\mu\nu a}[A]) Q_\nu^a$ in eq. (7) was absent for constant chromo-electric field E^a case in [3]. However, in the presence of time-dependent chromo-electric field $E^a(t)$ this single gluon term in eq. (7) is not zero. Hence the path integration in eq. (9) becomes more complicated due to the presence of this single gluon term.

To evaluate the path integration in eq. (9) we proceed as follows. We write

$$M_{\mu\nu}^{ab}(x, x') = \delta^{(4)}(x - x') M_{\mu\nu}^{ab}(x') \tag{12}$$

where we denote $M_{\mu\nu}^{ab}(x) = M_{\mu\nu}^{ab}[A](x)$ which is given by eq. (8). The Green's function $G_{\mu\nu}^{ab}(x, x')(= [M^{-1}]_{\mu\nu}^{ab}(x, x'))$ is given by

$$\int d^4 x'' M^{\mu\lambda, ac}(x, x'') G_{\lambda\nu}^{cb}(x'', x') = \delta^{ab} g_\nu^\mu \delta^{(4)}(x - x'). \tag{13}$$

We change the variable

$$Q_\mu^a(x) = Q_\mu'^a(x) - \frac{1}{2} \int d^4 x' G_{\mu\nu}^{ab}(x, x') D_\lambda(x') F^{\lambda\nu b}(x') \tag{14}$$

where we denote $D_\mu^{ab}(x) = D_\mu^{ab}[A](x)$ and $F_{\mu\nu}^a(x) = F_{\mu\nu}^a[A](x)$. Under this change of variable $[dQ] = [dQ']$. Using eqs. (14), (13) and (12) we find

$$[(D_\mu[A] F^{\mu\nu a}[A]) Q_\nu^a + Q^{\mu a} M_{\mu\nu}^{ab}[A] Q^{\nu b}]$$
$$= -\frac{1}{2} \int d^4 x' D_\mu(x) F^{\mu\lambda a}(x) G_{\mu\nu}^{ab}(x, x') D_\sigma(x') F^{\sigma\nu b}(x') + Q'^{\mu a} M_{\mu\nu}^{ab}[A] Q'^{\nu b} \tag{15}$$

Hence we find from eq. (9)

$$< 0|0 >^A = \frac{Z[A]}{Z[0]} = \exp[-\frac{i}{2} \int d^4 x \int d^4 x' D_\mu(x) F^{\mu\lambda a}(x) G_{\mu\nu}^{ab}(x, x') D_\sigma(x') F^{\sigma\nu b}(x')]$$
$$\times \frac{\int [dQ'] e^{i \int d^4 x Q'^{\mu a} M_{\mu\nu}^{ab}[A] Q'^{\nu b}}}{\int [dQ] e^{i \int d^4 x Q^{\mu a} M_{\mu\nu}^{ab}[0] Q^{\nu b}}} = e^{-iS_{\text{tad}}} \times \frac{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[A]}{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[0]} = e^{-iS_{\text{tad}}} \times e^{iS^{(1)}} \tag{16}$$

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where

$$S_{\text{tad}} = \frac{1}{2} \int d^4x \int d^4x' D_\mu(x) F^{\mu\lambda a}(x) G_\lambda^{\nu ab}(x,x') D^\sigma(x') F_{\sigma\nu b}(x') \tag{17}$$

is the tadpole (or single gluon) effective action and

$$S^{(1)} = -i\ln\left[\frac{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[A]}{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[0]}\right] \tag{18}$$

is the one loop (or gluon pair) effective action.

A. Tadpole (or Single Gluon) Effective Action in Arbitrary $E^a(t)$

For an operator M we will use Schwinger's notation [1] for the Green's function

$$G(x,x') = <x|\frac{1}{M}|x'> = <x|\int_0^\infty ds\; e^{-sM}|x'> . \tag{19}$$

The Green's function $G_{\mu\nu}^{ab}(x,x')$ for the operator $M_\mu^{\nu ab}[A]$ becomes

$$G_\mu^{\nu ab}(x,x') = <x|\int_0^\infty ds\; e^{-sM_\mu^{\nu ab}[A]}|x'> . \tag{20}$$

Using eq. (10) in (8) we find

$$M_\mu^{\nu ab}[A] = M_{\mu\lambda}^{ab}[A]g^{\lambda\nu} = -\delta_\mu^\nu[(\delta^{ab}\hat{p}_0 - igf^{abc}E^c(t)z)^2 - \delta^{ab}\hat{p}_z^2 - \delta^{ab}\hat{p}_T^2] - 2gf^{abc}E^c(t)\hat{F}_\mu^\nu \tag{21}$$

where

$$\hat{F}_\mu^\nu = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

By using eq. (21) in (20) we find

$$G_\mu^{\nu ab}(x,x') = <x|\int_0^\infty ds\; e^{-s(\delta_\mu^\nu[-(\delta^{ab}\hat{p}_0 - igf^{abc}E^c(t)z)^2 + \delta^{ab}\hat{p}_z^2 + \delta^{ab}\hat{p}_T^2] - 2gf^{abc}E^c(t)\hat{F}_\mu^\nu}|x'> . \tag{23}$$

We write eq. (23) in the Lorentz and color matrix notation as follows

$$G_\mu^{\nu,ab}(x,x') = \int_0^\infty ds[<x|\int_0^\infty ds\; e^{-s(-(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2 + 2ig\Lambda(t)\hat{F})}|x'>]_\mu^{\nu ab} \tag{24}$$

where

$$\Lambda^{ab}(t) = if^{abc}E^c(t). \tag{25}$$

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Using eqs. (24) and (11) in (17) we find the tadpole effective action

$$S_{\text{tad}} = \frac{1}{2} \int d^4x d^4x' \int_0^\infty ds \, \frac{dE^a(t)}{dt}[< x|e^{-s(-(\hat{p}_0 - ig\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2 + \, 2ig\Lambda(t)\hat{F})}|x' >]_3^{3ab} \frac{dE^b(t')}{dt'}$$

$$= \frac{1}{2} \int d^2x_T \, d^2x'_T \, dt \, dz \, dt' \, dz' \int_0^\infty ds$$

$$\frac{dE^a(t)}{dt}[< t| < z| < x_T|e^{-s(-(\hat{p}_0 - ig\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2 + \, 2ig\Lambda(t)\hat{F})}|x'_T > |z' > |t' >]_3^{3ab} \frac{dE^b(t')}{dt'}. \quad (26)$$

Inserting complete set of $|p_T>$ states (by using $\int d^2p_T |p_T><p_T| = 1$) we find

$$S_{\text{tad}} = \frac{1}{2} \int d^2x_T \, d^2x'_T \, dt \, dz \, dt' \, dz' \, d^2p_T \int_0^\infty ds \, \frac{dE^a(t)}{dt}[< x_T|p_T >$$

$$< t| < z| \, e^{-s(-(\hat{p}_0 - ig\Lambda(t)z)^2 + \hat{p}_z^2 + p_T^2 + \, 2ig\Lambda(t)\hat{F})}|t' > |z' >< p_T|x'_T >]_3^{3ab} \frac{dE^b(t')}{dt'}. \quad (27)$$

Using $< q|p >= \frac{1}{\sqrt{2\pi}}e^{iqp}$ we obtain

$$S_{\text{tad}} = \frac{1}{2(2\pi)^2} \int d^2x_T \, d^2x'_T \, dt \, dz \, dt' \, dz' \, d^2p_T \int_0^\infty ds \, \frac{dE^a(t)}{dt}[e^{ix_T \cdot p_T}$$

$$< t| < z|e^{-s(-(\hat{p}_0 - ig\Lambda(t)z)^2 + \hat{p}_z^2 + p_T^2 + \, 2ig\Lambda(t)\hat{F})}|z' > |t' > e^{-ix'_T \cdot p_T}]_3^{3ab} \frac{dE^b(t')}{dt'}. \quad (28)$$

Integrating over x'_T (by using $\int d^2x'_T e^{-ix'_T \cdot p_T} = (2\pi)^2 \, \delta^{(2)}(\vec{p}_T)$) we find

$$\frac{dS_{\text{tad}}}{d^4x d^2p_T} = \frac{1}{2} \, \delta^{(2)}(\vec{p}_T) \, e^{ix_T \cdot p_T} \int dt' \int dz' \int_0^\infty ds$$

$$\frac{dE^a(t)}{dt}[< t| < z|e^{-s(-(\hat{p}_0 - ig\Lambda(t)z)^2 + \hat{p}_z^2 + p_T^2 + \, 2ig\Lambda(t)\hat{F})}|z' > |t' >]_3^{3ab} \frac{dE^b(t')}{dt'}.$$

$$(29)$$

Since the above equation contains Dirac-delta function $\delta^{(2)}(\vec{p}_T)$ we find (for any non-vanishing p_T)

$$\frac{dS_{\text{tad}}}{d^4x d^2p_T} = 0. \quad (30)$$

Hence the tadpole (or single gluon) effective action does not contribute to the exact result $(\frac{dW}{d^4x d^2p_T})$ for the probability of gluon (pair) production per unit time per unit volume per unit transverse momentum from an arbitrary $E^a(t)$ via Schwinger mechanism.

Now we evaluate the one-loop effective action for gluon and ghost in the presence of arbitrary $E^a(t)$ in the following.

B.   One Loop (or Gluon Pair) Effective Action in Arbitrary $E^a(t)$

The one loop (or gluon pair) effective action eq. (18) can be written as

$$S^{(1)} = -i\ln[\frac{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[A]}{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[0]}] = \frac{i}{2}\text{Tr}[\ln M_{\mu}^{\nu ab}[A] - \ln M_{\mu}^{\nu ab}[0]]$$

$$= \frac{i}{2}\text{Tr}\int_0^\infty \frac{ds}{s}[e^{is(M_{\mu}^{\nu ab}[A]+i\epsilon)} - e^{is(M_{\mu}^{\nu ab}[0]+i\epsilon)}]. \tag{31}$$

The trace Tr is given by

$$\text{Tr}\mathcal{O} = \text{tr}_{\text{Lorentz}}\text{tr}_{\text{color}}\int d^4x < x|\mathcal{O}|x > . \tag{32}$$

Using eq. (21) in (31) and writing in matrix notations we find

$$S^{(1)} = \frac{i}{2}\text{tr}_{\text{Lorentz}}\text{tr}_{\text{color}}\int d^4x \int_0^\infty \frac{ds}{s}[< x|e^{-is((\hat{p}_0-g\Lambda(t)z)^2-\hat{p}_z^2-\hat{p}_T^2- 2ig\Lambda(t)\hat{F}-i\epsilon)} - e^{-is(\hat{p}^2-i\epsilon)}|x >]_{\mu}^{\nu,ab} \tag{33}$$

where the Lorentz matrix $\hat{F}_{\mu}^{\nu}$ and the color matrix Λ^{ab} matrices are given by eqs. (22) and (25) respectively.

Using the eigen values

$$\hat{F}_{\text{eigenvalues}} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (1, 0, -1, 0). \tag{34}$$

we perform the Lorentz trace and find

$$S^{(1)} = \frac{i}{2}\sum_{l=1}^4 \text{tr}_{\text{color}}\,[\int_0^\infty \frac{ds}{s}\int dt < t|\int dx < x|\int dy < y|\int dz < z|$$

$$e^{-is((\hat{p}_0-g\Lambda(t)z)^2-\hat{p}_z^2-\hat{p}_T^2- 2i\lambda_l g\Lambda(t)-i\epsilon)} - e^{-is(\hat{p}^2-i\epsilon)}|z > |y > |x > |t >]^{ab}. \tag{35}$$

Inserting complete set of $|p_T >$ states (using $\int d^2 p_T\ |p_T >< p_T| = 1$) we find from the above equation

$$S^{(1)} = \frac{i}{2(2\pi)^2}\sum_{l=1}^4 \text{tr}_{\text{color}}[\int_0^\infty \frac{ds}{s}\int d^2 x_T \int d^2 p_T e^{is(p_T^2+i\epsilon)}$$

$$[\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dz < z|e^{-is[(i\frac{d}{dt}-g\Lambda(t)z)^2-\hat{p}_z^2-2ig\lambda_l\Lambda(t)]}|z > |t > - \int dt \int dz \frac{1}{4\pi s}]]^{ab} \tag{36}$$

where we have used the normalization $< q|p >= \frac{1}{\sqrt{2\pi}}e^{iqp}$. At this stage we use the shift theorem [10] and find

$$S^{(1)} = \frac{i}{2(2\pi)^2}\sum_{l=1}^4 \text{tr}_{\text{color}}[\int_0^\infty \frac{ds}{s}\int d^2 x_T \int d^2 p_T e^{is(p_T^2+i\epsilon)}[\int_{-\infty}^{+\infty} dt < t|\int_{-\infty}^{+\infty} dz$$

$$< z + \frac{i}{g\Lambda(t)} \frac{d}{dt} |e^{-is[g^2\Lambda^2(t)z^2 - \hat{p}_z^2 - 2ig\lambda_l\Lambda(t)]}|z + \frac{i}{g\Lambda(t)} \frac{d}{dt} > |t > - \int dt \int dz \frac{1}{4\pi s}]]^{ab} \tag{37}$$

where the z integration must be performed from $-\infty$ to $+\infty$ for the shift theorem to be applicable.

Note that a state vector $|z + \frac{i}{a(t)} \frac{d}{dt} >$ which contains derivative operator is not familiar in physics. However, the state vector $|z + \frac{i}{a(t)} \frac{d}{dt} >$ contains the derivative $\frac{d}{dt}$ not $\frac{d}{dz}$. Hence the state vector is defined in the z-space with $\frac{d}{dt}$ acting as a c-number shift in z. To see how one operates with such state vector we find

$$< z + \frac{i}{a(t)} \frac{d}{dt} |p_z > f(t) = \frac{1}{\sqrt{2\pi}} e^{i(z + \frac{i}{a(t)} \frac{d}{dt})p_z} f(t) = \frac{1}{\sqrt{2\pi}} e^{izp_z} e^{-\frac{p_z}{a(t)} \frac{d}{dt}} f(t). \tag{38}$$

Inserting complete sets of $|p_z >$ states (using $\int dp_z \, |p_z >< p_z| = 1$) in eq. (37) we find

$$S^{(1)} = \frac{i}{2(2\pi)^2} \sum_{l=1}^{4} \int_0^\infty \frac{ds}{s} \int d^2 x_T \int d^2 p_T e^{is(p_T^2 + i\epsilon)} [F_l(s) - \int dt \int dz \frac{8}{4\pi s}] \tag{39}$$

where

$$F_l(s) = \frac{1}{(2\pi)} \text{tr}_{\text{color}} \left[\int_{-\infty}^{+\infty} dt < t| \int dp_z \int dp_z' \int_{-\infty}^{+\infty} dz \, e^{izp_z} e^{-\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z} \right.$$
$$\left. < p_z |e^{is[-g^2\Lambda^2(t)z^2 + \hat{p}_z^2 + 2i\lambda_l g\Lambda(t)]}|p_z' > e^{\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z'} e^{-izp_z'} |t >]^{ab}. \tag{40}$$

It can be seen that the exponential $e^{-\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z}$ contains the derivative $\frac{d}{dt}$ which operates on $< p_z |e^{is[-g^2\Lambda^2(t)z^2 + \hat{p}_z^2 - i2\lambda_l g\Lambda(t)]}|p_z' >$ hence we can not move $e^{-\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z}$ to right. We insert more complete sets of states to find

$$F_l(s) = \frac{1}{(2\pi)} \text{tr}_{\text{color}} \left[\int_{-\infty}^{+\infty} dt \int dt' \int dt'' \int dz' \int dz'' \int_{-\infty}^{+\infty} dz \int dp_0 \int dp_0' \int dp_0'' \int dp_0''' \right.$$
$$\int dp_z \int dp_z' < t|p_0 > e^{ip_z} < p_0 e^{-\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z}|p_0' >< p_0'|t' >< t'| < p_z|z' >< z'|$$
$$e^{is[-g^2\Lambda^2(t)z^2 + \hat{p}_z^2 + 2i\lambda_l g\Lambda(t)]}|z'' >< z''|p_z' > |t'' >< t''|p_0'' >< p_0''|e^{\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z'}|p_0''' > e^{-izp_z'} < p_0'''|t >]^{ab}$$
$$= \frac{1}{(2\pi)^4} \text{tr}_{\text{color}} \left[\int_{-\infty}^{+\infty} dt \int dt' \int dz' \int dz'' \int_{-\infty}^{+\infty} dz \int dp_0 \int dp_0' \int dp_0'' \int dp_0''' \right.$$
$$\int dp_z \int dp_z' e^{itp_0} e^{ip_z} < p_0 e^{-\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z}|p_0' > e^{-it'p_0'} e^{iz'p_z}$$
$$< z'|e^{is[-g^2\Lambda^2(t')z^2 + \hat{p}_z^2 + 2i\lambda_l g\Lambda(t')]}|z'' > e^{iz''p_z'} e^{it'p_0''} < p_0''|e^{\frac{1}{g\Lambda(t)} \frac{d}{dt} p_z'}|p_0''' > e^{-izp_z'} e^{-itp_0'''}]^{ab}. \tag{41}$$

It can be seen that all the expressions in the above equation are independent of t except $e^{it(p_0 - p_0''')}$. This can be seen as follows

$$< p_0|f(t) \frac{d}{dt}|p_0' >= \int dt' \int dt'' \int dp_0'''' < p_0|t' >< t'|f(t)|t'' >< t''|p_0'''' >< p_0''''| \frac{d}{dt}|p_0' >$$

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$$= \int dt' \int dt'' \int dp_0'''' e^{-it'p_0} \, \delta(t'-t'')f(t'')e^{it''p_0''''} \, ip_0' \, \delta(p_0''''-p_0') = ip_0' \int dt' \; e^{-it'(p_0-p_0')}f(t')$$

(42)

which is independent of t and $\frac{d}{dt}$. Hence by using the cyclic property of trace we can take the matrix $[<p_0''|e^{\frac{1}{g\Lambda(t)}\frac{d}{dt}p_z}|p_0'''>]^{ab}$ to the left. The t integration is now easy $(\int_{-\infty}^{+\infty} dt e^{it(p_0-p_0''')} = 2\pi\delta(p_0-p_0'''))$ which gives

$$F_l(s) = \frac{1}{(2\pi)^3}\text{tr}_{\text{color}} \; [\int dt' \int dz' \int dz'' \int_{-\infty}^{+\infty} dz \int dp_0 \int dp_0' \int dp_0'' \int dp_z \int dp_z' e^{itp_0}e^{izp_z}$$
$$<p_0''|e^{\frac{1}{g\Lambda(t)}\frac{d}{dt}p_z'}|p_0> <p_0e^{-\frac{1}{g\Lambda(t)}\frac{d}{dt}p_z}|p_0'> \; e^{-iz'p_z}e^{-it'p_0'} <z'|e^{is[-g^2\Lambda^2(t')z^2+\hat{p}_z^2+2i\lambda_l g\Lambda(t')]}|z''>$$
$$e^{it'p_0''}e^{iz''p_z'}e^{-izp_z'}]^{ab}.$$

(43)

As advertised earlier we must integrate over z from $-\infty$ to $+\infty$ for the shift theorem to be applicable [10]. The matrix element $<z'|e^{is[-g^2\Lambda^2(t)z^2+\hat{p}_z^2+i2\lambda_l g\Lambda(t)]}|z''>$ is independent of z variable (it depends on z' and z'' variables). Hence we can perform the z integration easily by using $\int_{-\infty}^{+\infty} dz e^{iz(p_z-p_z')} = 2\pi\delta(p_z-p_z')$ to find

$$F_l(s) = \frac{1}{(2\pi)^2}\text{tr}_{\text{color}} \; [\int dt' \int dz' \int dz'' \int dp_0 \int dp_0' \int dp_0'' \int dp_z e^{itp_0}$$
$$<p_0''|e^{\frac{1}{g\Lambda(t)}\frac{d}{dt}p_z}|p_0> <p_0e^{-\frac{1}{g\Lambda(t)}\frac{d}{dt}p_z}|p_0'> \; e^{-iz'p_z}e^{-ip_0't'} <z'|e^{is[-g^2\Lambda^2(t')z^2+\hat{p}_z^2+2i\lambda_l g\Lambda(t')]}|z''>$$
$$e^{p_0''t'}e^{iz''p_z}]^{ab}.$$

(44)

Using the completeness relation $\int dp_0|p_0><p_0| = 1$ we obtain

$$F_l(s) = \frac{1}{(2\pi)^2}\text{tr}_{\text{color}} \; [\int dt' \int dz' \int dz'' \int dp_0' \int dp_z$$
$$e^{-iz'p_z} <z'|e^{is[-g^2\Lambda^2(t')z^2+\hat{p}_z^2+2i\lambda_l g\Lambda(t')]}|z''> e^{iz''p_z}]^{ab}.$$

(45)

Since $<z'|e^{is[-g^2\Lambda^2(t')z^2+\hat{p}_z^2+2i\lambda_l g\Lambda(t')]}|z''>$ is independent of z variable (it depends on z' and z'' variables) we can integrate over p_z. We find (by using $\int dp_z e^{i(z''-z')p_z} = (2\pi)\delta(z'-z'')$)

$$F_l(s) = \frac{1}{(2\pi)}\text{tr}_{\text{color}} \; [\int dt \int dp_0 \int dz' <z'|e^{is[-g^2\Lambda^2(t)z^2+\hat{p}_z^2+2i\lambda_l g\Lambda(t)]}|z'>]^{ab}.$$

(46)

The Lorentz force equation in color space (in the adjoint representation of SU(3)) is given by $\delta^{ab}dp_\mu = gT_{ab}^c F_{\mu\nu}^c dx^\nu = igf^{abc}F_{\mu\nu}^c dx^\nu$. When the chromo-electric field is along the z-axis (eq. (10)) this becomes $\delta^{ab}dp_0 = igf^{abc}E^c(t)dz = g\Lambda^{ab}(t)dz$. Using this in (46) and using the eigen values of the color matrix $\Lambda^{ab}(t)$ [2] from eq. (2) we find

$$F_l(s) = \frac{1}{(2\pi)}\sum_{j=1}^{6} \int dt \; g\Lambda_j(t) \int dz \int dz' <z'|e^{is[-g^2\Lambda_j^2(t)z^2+\hat{p}_z^2+2i\lambda_l g\Lambda_j(t)]}|z'> .$$

(47)

The above equation boils down to an usual harmonic oscillator, $\omega^2(t)z'^2 + \hat{p}_{z'}^2$, with the constant frequency ω replaced by time dependent frequency $\omega(t)$. The normalized wave function is given by [4]

$$\int dz' |<z'|n_t>|^2 = 1. \tag{48}$$

Inserting complete set of harmonic oscillator states (by using $\sum_n |n_t><n_t| = 1$) in eq. (47) we find

$$F_l(s) = \frac{1}{(2\pi)^2} \sum_n \sum_{j=1}^6 \int dt \ g\Lambda_j(t) \int dz \int dz' <z'|n_t> e^{-sg\Lambda_j(t)(2n+1)+2\lambda_l g\Lambda_j(t)}$$

$$<n_t|z'> = \frac{1}{(2\pi)} \sum_n \sum_{j=1}^6 \int dt \ g\Lambda_j(t) \int dz \int dz' |<z'|n_t>|^2 e^{-sg\Lambda_j(t)(2n+1)+2\lambda_l g\Lambda_j(t)}$$

$$= \frac{1}{(2\pi)} \sum_{j=1}^6 \int dt \int dz \ g\Lambda_j(t) \frac{e^{sg2\lambda_l\Lambda_j(t)}}{2\mathrm{sinh}(sg\Lambda_j(t))} \tag{49}$$

where we have used eq. (48). Using this expression of $F_l(s)$ in eq. (39) and summing over l (by using the eigen values of the Dirac matrix from eq. (34)) we find

$$S^{(1)} = \frac{i}{16\pi^3} \sum_{j=1}^6 \int_0^\infty \frac{ds}{s} \int d^4x \int d^2p_T e^{is(p_T^2+i\epsilon)} [g\Lambda_j(t) \frac{[1+\cosh(2sg\Lambda_j(t))]}{\sinh(sg\Lambda_j(t))} - \frac{2}{s}]. \tag{50}$$

C.   One Loop Effective Action for Ghost in arbitrary $E^a(t)$

Now we discuss the ghost contributions. The ghost Lagrangian density due to the gauge fixing term in eq. (5) is given by [8, 9]

$$\mathcal{L}_{\mathrm{gh}} = \chi^{a\dagger} [D_\mu[A]D^\mu[A+Q]]^{ab}\chi^b = \chi^{a\dagger} K^{ab}[A,Q]\chi^b \tag{51}$$

where χ^a is the ghost field. The vacuum-to-vacuum transition amplitude for ghost (anti-ghost) in the presence of $A_\mu^a(x)$ is given by:

$$<0|0>^A = \frac{Z[A]}{Z[0]} = \frac{\int [d\chi^\dagger][d\chi]e^{i\int d^4x \chi^{\dagger a} K^{ab}[A]\chi^b}}{\int [d\chi^\dagger][d\chi]e^{i\int d^4x \chi^{\dagger a} K^{ab}[0]\chi^b}} = \frac{\mathrm{Det}K^{ab}[A]}{\mathrm{Det}K^{ab}[0]} = e^{iS_{\mathrm{gh}}^{(1)}}. \tag{52}$$

This gives

$$S_{\mathrm{gh}}^{(1)} = -i\mathrm{ln}[\frac{\mathrm{Det}K^{ab}[A]}{\mathrm{Det}K^{ab}[0]}] = -i\mathrm{Tr}[\mathrm{ln}K^{ab}[A] - \mathrm{ln}K^{ab}[0]] \tag{53}$$

where $K^{ab}[A] = K^{ab}[A, Q=0]$ is given by eq. (51). We can repeat the ghost calculation similar to the gluon except for the following changes. There is an over all factor $\frac{1}{4}$ (with

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$[1 + \cosh(2g\Lambda_j(t))] \rightarrow 2)$ from eq. (50) because we do not have any Lorentz matrices in $K^{ab}[A]$ in eq. (51). There is another over all factor (-2) from eq. (50) because of the ghost determinant (compare eqs. (18) and (53)). With these two changes we find from eq. (50) the following expression for the ghost one-loop effective action

$$S_{\text{gh}}^{(1)} = -\frac{i}{16\pi^3} \sum_{j=1}^{6} \int_0^\infty \frac{ds}{s} \int d^4x \int d^2p_T e^{is(p_T^2+i\epsilon)} \Big[\frac{g\Lambda_j(t)}{\sinh(sg\Lambda_j(t))} - \frac{1}{s} \Big]. \tag{54}$$

D.   Non-Perturbative Gluon Pair Production From Arbitrary $E^a(t)$ via Schwinger Mechanism

Adding eqs. (50) and (54) for the effective action for gluon and ghost respectively we find

$$S_{\text{gl}}^{(1)} = \frac{i}{16\pi^3} \sum_{j=1}^{6} \int_0^\infty \frac{ds}{s} \int d^4x \int d^2p_T e^{is(p_T^2+i\epsilon)} \Big[g\Lambda_j(t) \frac{\cosh(2sg\Lambda_j(t))}{\sinh(sg\Lambda_j(t))} - \frac{1}{s} \Big]. \tag{55}$$

The imaginary part of the above effective action gives real gluon pair production. The real part of the above equation is infrared divergent as $s \rightarrow \infty$. However we are interested in the imaginary part of the above effective action which is not infrared divergent as $s \rightarrow \infty$. This can be easily checked by making $s \rightarrow is$. The s-contour integration can be done in the similar way as was done in [1, 3, 4, 11]. Using the series expansion

$$\frac{1}{\sinh x} = \frac{1}{x} + 2x \sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^2 n^2 + x^2} \tag{56}$$

we perform the s-contour integration around the pole $s = \frac{in\pi}{|g\Lambda_j(t)|}$ to find

$$W = 2\text{Im}S_{\text{gl}}^{(1)} = \frac{1}{4\pi^3} \sum_{j=1}^{3} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \int d^4x \int d^2p_T |g\Lambda_j(t)| e^{-\frac{n\pi p_T^2}{|g\Lambda_j(t)|}}. \tag{57}$$

Hence the probability of non-perturbative gluon (pair) production per unit time, per unit volume and per unit transverse momentum from an arbitrary time dependent chromo-electric field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3) is given by

$$\frac{dW}{dt d^3x d^2p_T} = \frac{1}{4\pi^3} \sum_{j=1}^{3} |g\Lambda_j(t)| \ln[1 + e^{-\frac{\pi p_T^2}{|g\Lambda_j(t)|}}], \tag{58}$$

which reproduces eq. (1). The expressions for gauge invariant $\Lambda_j(t)$'s are given in eq. (2).

III. CONCLUSION

To conclude we have studied Schwinger mechanism for gluon pair production in the presence of an arbitrary time-dependent chromo-electric background field $E^a(t)$ with arbitrary color index a=1,2,...8 in SU(3). We have obtained an exact result for the probability of non-perturbative gluon (pair) production per unit time per unit volume per unit transverse momentum $\frac{dW}{d^4x d^2p_T}$ from arbitrary $E^a(t)$ by directly evaluating the path integral. We have found that the tadpole (or single gluon) effective action does not contribute to the non-perturbative gluon pair production rate $\frac{dW}{d^4x d^2p_T}$. We have found that the exact result for non-perturbative gluon pair production is independent of all the time derivatives $\frac{d^n E^a(t)}{dt^n}$ where $n = 1, 2, ...\infty$ and has the same functional dependence on two casimir invariants $[E^a(t)E^a(t)]$ and $[d_{abc}E^a(t)E^b(t)E^c(t)]^2$ as the constant chromo-electric field E^a result with the replacement:$E^a \rightarrow E^a(t)$. This result relies crucially on the validity of the shift conjecture, which has not yet been established. This result may be relevant to study the production of a non-perturbative quark-gluon plasma at RHIC and LHC [5, 6, 7].

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